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## **FLEET OPTIMIZATION FOR A SELECTIVE SOLID WASTE COLLECTION SYSTEM**

### **ABSTRACT**

**Background:** In the paper the vehicle routing problem (VRP) combined with fleet optimization for selected solid waste collection is presented. The fleet of garbage trucks consists of vehicles of various types which can differ one from another with capacity, size, and exclusive assignment to certain types of waste. Garbage trucks are used to collect segregated solid waste from pickup nodes and transport it to specialized sorting units. In real selective solid waste collection systems, an important issue is to route garbage truck subject to not only their size but also the time windows of pickup nodes. Some solid waste drop-off points are located in narrow streets and they can be served only by small-sized or medium-sized garbage trucks. Some pickup nodes can be served only within predefined time windows and visiting them should be scheduled carefully, so that they can be served in the preferred period and garbage trucks' extra dwell time or extra kilometrage can be avoided.

**Methods:** The Fleet Optimization Problem for Selective Solid Waste Collection (FOPSSWC) continues the approach adopted for the Selective Solid Waste Collection Routing Problem (SSWCRP). The SSWCRP solves the VRP for a predefined heterogeneous fleet of garbage trucks, includes time windows of pickup nodes, and recognizes segregated waste types to be served by dedicated garbage trucks. The FOPSSWC finds the minimum number of garbage

trucks needed to perform selective waste collection service and not to violate any constraint. For the FOPSSWC a Mixed Integer Program was formulated and solved using Gurobi solver.

**Results:** Gurobi solver was employed for solving small-sized and medium-sized instances. Results obtained for exact optimization of the Fleet Optimization Problem for Selective Solid Waste Collection (FOPSSWC) are illustrated with an exemplary instance for which computational experiments are presented and obtained results are reported.

**Conclusions:** The main contribution of the paper is a newly developed Mixed Integer Program combining fleet optimization with the VRP for the selective solid waste collection system. The FOPSSWC may be used by Solid Waste Management for scheduling selective solid waste collection. Obtained schedules may result in reducing the total operational costs of periodical selective solid waste collection.

**Keywords:** selective solid waste collection system, MILP, rich VRP, Solid Waste Management

## INTRODUCTION

The public concern for environmental preservation makes Solid Waste Management (SWM) one of the modern society's most relevant issues. Improperly managed municipal solid waste contaminates oceans, clogs drains and causes flooding, transmits diseases, and harms animals that consume waste unknowingly. Moreover, issues due to the inadequate waste collection and disposal systems are the main factors for the spread of serious diseases in cities. Collection and disposal of municipal solid waste is a stinging and widespread problem in both urban and rural areas in developed and developing countries. A well-designed SWM system can interconnect all aspects of solid waste streams, solid waste collection, and solid waste processing, such that the optimum values of the environmental and socioeconomic objectives have been achieved concurrently. Therefore, SWM may be considered as a way to mitigate the impact of climate change [Ghiani et al. 2014, Edalatpour et al. 2018].

The fleet of garbage trucks consists of vehicles which can differ one from another with capacity, size, and exclusive assignment to certain types of solid waste. Garbage trucks are used to collect segregated solid waste from pickup nodes and transport it to specialized sorting units. In real selective solid waste collection systems, an important issue is to route garbage truck subject to their size and the time windows of pickup nodes. Some solid waste drop-off points are located in narrow streets and they can be served only by small-sized or medium-sized garbage trucks. Some pickup nodes can be served only within predefined time windows and visiting them should

be scheduled carefully, so that they are served in the preferred period and garbage trucks' extra dwell time or extra kilometrage is avoided. In this paper a vehicle routing problem (VRP) combined with fleet optimization for selected solid waste collection is presented.

Fleet Optimization Problem for Selective Solid Waste Collection (FOPSSWC) continues the approach adopted for Selective Solid Waste Collection Routing Problem (SSWCRP) [Korcyl et al. 2015, Korcyl et al. 2016]. The SSWCRP solves the VRP with time windows for pickup nodes with a predefined heterogeneous fleet of garbage trucks and recognizes segregated solid waste types to be served by garbage trucks of each type. The FOPSSWC finds among others the minimum number of garbage trucks needed to perform selective waste collection service subject to all the constraints. For the FOPSSWC a Mixed Integer Program was formulated and solved using Gurobi solver.

The reminder of the paper is organized as follows. The next section provides a short literature review on Solid Waste Management and the VRP for SWM. Then the newly developed MILP model for the FOPSSWC is presented and illustrated with an exemplary instance for which computational experiments are presented and obtained results are reported.

## **SELECTIVE SOLID WASTE MANAGEMENT IN POLAND**

In 2018 Polish Ministry of Environment surveyed how Polish citizens were dealing with municipal solid waste sorting. The report showed that ca. 16% of the owners of real estates did not segregate solid waste generated in their properties [Ministerstwo Środowiska 2019]. According to the data published by the Polish Statistical Office on June 30, 2018, Polish population was 38 413 000 [Główny Urząd Statystyczny 2018], which means that 6 146 080 Polish citizens did not segregate municipal solid waste. The avoidance of solid waste segregation can result in the increase of the total costs of Solid Waste Management due to three factors: (i) the need for unsegregated solid waste disposal and treatment, (ii) the waste of materials that could be recycled and reused, and (iii) environmental contamination. The increase of the total costs of SWM will influence the municipal solid waste charging scheme, which means that waste fee will rise.

The latest amendment to so-called Law on Waste [Dz.U. 2019 poz. 1579, 2019] introduced differentiation of fees for municipal solid waste treatment depending on the type of solid waste. The amendment is supposed to engage citizens to segregate solid waste: citizens segregating their solid waste would be offered discounted waste fee. According to the amendment, municipalities may set waste fee for segregated solid waste as at least the half of waste fee for non-segregated solid waste, while maximum fee for non-segregated solid waste must not

exceed four times the fee for segregated solid waste. Municipalities can compensate offered discounts with incomes obtained from sales of segregated recyclable materials.

Municipalities are free to choose the way of municipal solid waste collection, so different SWM systems may be introduced by different municipalities. Majority of municipalities use two selective solid waste collection systems. The first system consists of municipal solid waste collection at source – municipal solid waste is collected at solid waste producers’; this system is popular in single-family residential areas in agglomerations or in smaller cities (see Figure 1). In the second system municipal solid waste is collected at solid waste drop-off points – citizens deliver their solid waste to a shared drop-off point; this system works in big cities or agglomerations in areas where multi-apartment residential buildings dominate (see Figure 2) [Bilitewski et al. 2000].

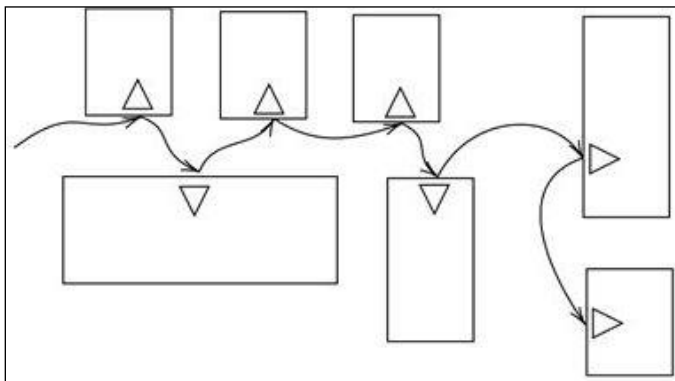


Fig. 1. Solid waste collection system at source.  
Source: [Bilitewski et al. 2000].

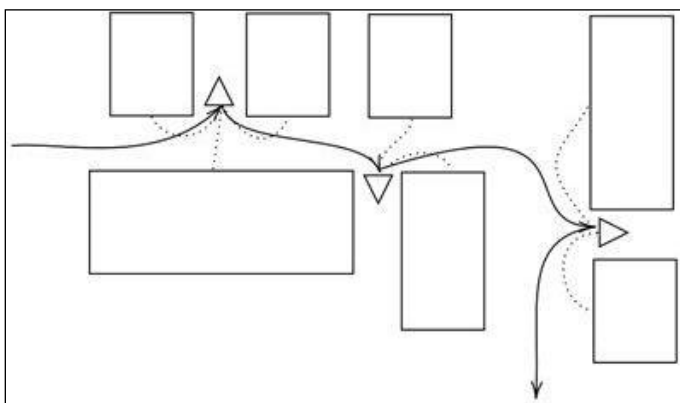


Fig. 2. Solid waste collection system at drop-off points.  
Source: [Bilitewski et al. 2000].

With reference to amendments to the Law on Waste the Polish Minister for Environment issued a new regulation which describes in details requirements for solid waste transport, means and ways of solid waste transportation. What is more, the Minister for Environment annulled

regionalization for municipal solid waste management, which means that nowadays it is allowed to transport non-segregated solid waste and remains after solid waste segregation to any solid waste landfill installation in Poland.

Nowadays, the main issue faced by municipalities is adjusting their solid waste collection system to solid waste segregation and treatment methods chosen by the municipal Solid Waste Management. The service provided by the SWM should be prompt, should fully comply with the respective legislation, and should minimize the total costs. Combined transport of segregated solid waste is very difficult due to restrictions and required using vocational vehicles. Municipalities have databases on segregated solid waste pickup points and amounts of solid waste to be collected from them, so they may organize tenders, so that they can choose professional companies to provide selective solid waste collection. Such companies need to adjust their fleet of vocational vehicles to the solid waste collection system adopted by the SWM. Therefore, optimization problems faced by such companies refer not only to determining fleet size but also to deciding on the types of vocational vehicles that should be included into the fleet.

### **VEHICLE ROUTING PROBLEM IN SOLID WASTE MANAGEMENT**

Here we briefly review rich VRPs for SWM to which we refer in the FOPSSWC, while detailed survey can be found in [Ghiani et al. 2014].

In [Xue et al. 2015] capacitated VRP is used as the point of reference for an allocation problem aiming at determining the amount of solid waste to be sent from each pickup point to each landfill to minimize overall transportation cost.

Buhrkal et al. [2012] developed the Waste Collection Vehicle Routing Problem with Time Window (WCVRPTW) where the objective was to find cost optimal routes for garbage trucks such that all garbage bins are emptied and waste is taken to landfills while respecting customer time windows and ensuring that drivers are given the breaks required by the law. The WCVRPTW differs from the traditional VRPTW by that the garbage trucks must visit landfills in order to empty their load.

In [Teixeira et al. 2004] the Periodic VRP (PVRP) was developed for a SWM system, with the aim at minimizing the total operations costs by setting optimal waste collection routes for every day of the month, to be repeated every month.

Oliveira, Simonetto and de Borenstein [2007] developed the multi-depot VRP ( $m$ -VRP) to solve vehicle allocation and routing problem, where the objective was to minimize the total

vehicle transportation costs. It was also guaranteed that minimum percentage of solid waste would be sent to each sorting unit.

Markov et al. [2016] addressed complex recyclable solid waste collection problem that extended the class of VRP with intermediate facilities by integrating a heterogeneous fixed fleet and a flexible assignment of destination depots.

In [Asefi et al. 2019] the Fleet Size and Mix Vehicle Routing Problem (VRP) to optimize the cost-effective Integrated Solid Waste Management system was investigated; the transportation cost in the entire SWM system and total deviation from the fair load allocation to transfer stations are minimized with a bi-objective Mixed-Integer Linear Programming (MILP) model. Also, Ayvaz-Cavdaroglu et al. [2019] used a multi-objective approach to minimize the total cost of selected periodical solid waste collecting increasing the recovery rate for recyclable materials depending on the source and condition of the material.

## THE FLEET OPTIMIZATION PROBLEM FOR SELECTIVE SOLID WASTE COLLECTION

The Fleet Optimization Problem for Selective Solid Waste Collection can be represented with set of nodes  $V$ , where node 0 stands for both the beginning and the end of each route, subset  $R$  consists of selective solid waste pickup places, and subset  $P$  represents specialized sorting units. Set  $W$  consists of solid waste types. Set  $C$  is the set of types of garbage trucks used in the system, while set  $K$  consists of solid waste collection trips.

The following notation is used:

### *Variables*

$x_{ijkw}$  – 1 if arc  $(i, j)$  belongs to waste collection trip  $k$  during which selective solid waste  $w$  is collected, 0 otherwise;

$z_{ijkc}$  – 1 if arc  $(i, j)$  which belongs to waste collection trip  $k$  is served by a garbage truck of type  $c$ , 0 otherwise;

$y_{kw}$  – 1 if solid waste  $w$  is to be collected during solid waste collection trip  $k$ , 0 otherwise;

$a_{ik}$  – arrival time at node  $i$  of a garbage truck serving trip  $k$ ;

$u_{ik}$  – dwell time in node  $i$  of a garbage truck serving trip  $k$ ;

$u'_{ikc}$  – dwell time in node  $i$  of a garbage truck of type  $c$  serving waste collection trip  $k$ ;

### *Parameters*

$d_{iw}$  – the amount of solid waste  $w$  to be collected from node  $i$ ;

$e_{iw}$  – the earliest time of picking up solid waste  $w$  from node  $i$ ;

- $l_{iw}$  – the latest time of picking up the solid waste  $w$  from node  $i$ ;
- $s_{iw}$  – the amount of time needed for picking up the solid waste  $w$  from node  $i$ ;
- $E_w$  – the earliest time to leave a depot for collecting solid waste of type  $w$ ;
- $L_w$  – the latest time to leave a depot for collecting solid waste  $w$ ;
- $p_c$  – the latest time for a garbage truck of type  $c$  to get back to its home depot;
- $q_c$  – capacity of a garbage truck of type  $c$ ;
- $\alpha_c$  – the fixed cost of using a garbage truck of type  $c$ ;
- $\beta_c$  – the unit cost of using a garbage truck of type  $c$ ;
- $\delta_c$  – the cost of additional dwell time of a garbage truck of type  $c$ ;
- $t_{ij}$  – travel time between pickup nodes  $i$  and  $j$ ;
- $f_{ijc}$  – 1 if a garbage truck of type  $c$  is allowed to traverse arc  $(i, j)$ , otherwise 0;
- $h_{cw}$  – 1 if a garbage truck of type  $c$  is allowed to collect solid waste of type  $w$ , otherwise 0;
- $\gamma_1$  – coefficient of the fixed cost of using a garbage truck;
- $\gamma_2$  – coefficient of the unit cost of using a garbage truck;
- $\gamma_3$  – coefficient of the cost of extra dwelling;
- $M$  – large positive constant;

and the associated Mixed Integer Program is written as:

$$\begin{aligned} \text{minimize } z = & \sum_{k \in K} \sum_{w \in W} (1 - y_{kw}) + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{w \in W} x_{ijkw} + \\ & - \gamma_1 \sum_{c \in C} \sum_{k \in K} \sum_{j \in V \setminus \{0\}} \sum_{w \in W} \alpha_c z_{0jkc} - \gamma_2 \sum_{c \in C} \sum_{k \in K} \sum_{i \in V \setminus \{0\}} \sum_{j \in V \setminus \{0\}} \sum_{w \in W} \beta_c z_{ijkc} + \\ & - \gamma_3 \sum_{c \in C} \sum_{k \in K} \sum_{i \in V \setminus \{0\}} \delta_c u'_{ikc} \end{aligned} \quad (1)$$

$$\sum_{j \in V} \sum_{k \in K} x_{ijkw} \leq 1, \quad i \in V \setminus \{0\}, w \in W, i \neq j \quad (2)$$

$$\sum_{j \in V \setminus \{0\}} x_{0jkw} = y_{kw}, \quad k \in K, w \in W \quad (3)$$

$$\sum_{i \in P} x_{i0kw} = y_{kw}, \quad k \in K, w \in W \quad (4)$$

$$\sum_{i \in V, i \neq j} x_{ijkw} = \sum_{i \in V, i \neq j} x_{jikw}, \quad j \in V, k \in K, w \in W \quad (5)$$

$$x_{ijkw} + x_{jikw} \leq 1, \quad i \in V \setminus \{0\}, j \in V, k \in K, w \in W, i \neq j \quad (6)$$

$$x_{iikw} = 0, \quad i \in V, k \in K, w \in W \quad (7)$$

$$\sum_{i \in V \setminus \{0\}} \sum_{j \in V \setminus \{0\}} x_{ijkw} \leq M y_{kw}, \quad k \in K, w \in W, i \neq j \quad (8)$$

$$x_{ijkw} \leq \sum_{c \in C} z_{ijkc}, \quad i \in V \setminus \{0\}, j \in V, k \in K, w \in W, i \neq j \quad (9)$$

$$\sum_{c \in C} z_{ijkc} \leq 1, \quad i \in V \setminus \{0\}, j \in V, k \in K, i \neq j \quad (10)$$

$$\sum_{i \in V, i \neq j} z_{ijkc} = \sum_{i \in V, i \neq j} z_{jikc}, \quad j \in V, k \in K, c \in C \quad (11)$$

$$\sum_{i \in V \setminus \{0\}} \sum_{i \in V} d_{iw} z_{ijkc} \leq q_c, \quad c \in C, k \in K, w \in W \quad (12)$$

$$a_{ik} + u_{ik} + \sum_{w \in W} s_{iw} y_{kw} + t_{ij} - a_{jk} \leq M(1 - z_{ijkc}), i \in V, j \in V/\{0\}, k \in K, c \in C, i \neq j \tag{13}$$

$$a_{jk} - u_{ik} - \sum_{w \in W} s_{iw} y_{kw} - t_{ij} - a_{ik} \leq M(1 - x_{ijkc}), i \in V, j \in V/\{0\}, k \in K, c \in C, i \neq j \tag{14}$$

$$\sum_{w \in W} e_{iw} \sum_{j \in V, i \neq j} x_{ijkw} \leq a_{ik} + u_{ik}, i \in V/\{0\}, k \in K \tag{15}$$

$$a_{ik} + u_{ik} \leq \sum_{w \in W} l_{iw} \sum_{j \in V, i \neq j} x_{ijkw}, i \in V/\{0\}, k \in K \tag{16}$$

$$E_w - a_{0k} - u_{0k} \leq M(1 - y_{kw}), k \in K, w \in W \tag{17}$$

$$a_{0k} + u_{0k} - L_w \leq M(1 - y_{kw}), k \in K, w \in W \tag{18}$$

$$a_{ik} + u_{ik} + \sum_{w \in W} s_{iw} y_{kw} \leq p_c, c \in C, k \in K, i \in V/\{0\} \tag{19}$$

$$\sum_{i \in V, j \in P, i \neq j} x_{ijkw} + \sum_{i \in V, j \in P, i \neq j} x_{jikw} = 2 * y_{kw}, k \in K, w \in W \tag{20}$$

$$\sum_{i \in R, j \in P} x_{ijkw} \geq y_{kw}, k \in K, w \in W \tag{21}$$

$$u_{ik} - u'_{ikc} \leq M(1 - \sum_{j \in V/\{0\}} z_{ijkc}), i \in V, k \in K, w \in W \tag{22}$$

$$\sum_{w \in W} y_{kw} \leq 1, k \in K \tag{23}$$

$$z_{ijkc} \leq f_{ijc}, i, j \in V/\{0\}, k \in K, c \in C, i \neq j \tag{24}$$

$$\sum_{j \in V/\{0\}} z_{0jkc} \leq h_{cw}, c \in C, w \in W, k \in K \tag{25}$$

$$a_{0k} = 0, k \in K \tag{26}$$

$$a_{ik} \geq 0, k \in K, i \in V \tag{27}$$

$$u_{ik} \geq 0, k \in K, i \in V \tag{28}$$

$$u'_{ikc} \geq 0, k \in K, i \in V, c \in C \tag{29}$$

$$x_{ijkw} \in \{0,1\}, i, j \in V, k \in K, w \in W \tag{30}$$

$$z_{ijkc} \in \{0,1\}, i, j \in V, k \in K, c \in C \tag{31}$$

$$y_{kw} \in \{0,1\}, k \in K, w \in W \tag{32}$$

Using the objective function (1) we can optimize following criteria: (i) minimizing the total number of trips on which solid waste of each kind is collected separately, (ii) maximizing the number of arcs  $(i, j)$  belonging to trip  $k$ , (iii) minimizing the total fixed costs of using garbage trucks in the system, and (iv) minimizing the total unit costs generated by used garbage trucks. The total fixed costs of using garbage trucks depend on how many garbage trucks of every type is used in the system to collect selective solid waste, while the total unit costs are proportional to the total distance driven by the fleet. Parameters  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  are used in the objective function, so that the impact of every cost category can be adjusted, and the costs can be scaled properly. Generally, parameters  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  should yield relatively small values so that the



number of trips set within the network served by the SWM system can be minimized, and the total number of traverses of arcs can be maximized in the same time.

Constraint (2) guarantees that every pickup node is served at most once by a garbage truck collecting selective solid waste  $w$ . Thanks to constraints (3) and (4) during trip  $k$  waste  $w$  is collected, while constraints (22) and (23) guarantee that trip  $k$  is assigned to collecting waste of exactly one type. Moreover, constraint (3) and (4) ensure that trip  $k$  must begin and end at the same depot.

A trip on which selective waste  $w$  is collected is defined by constraints (5), (6), and (7) as a sequence of pickup nodes which starts and ends at the same depot. Thanks to constraint (5) arc  $(i, j)$  belongs to exactly one trip on which waste  $w$  is collected.

Constraints (8), (9), (10), and (11) guarantee that arc  $(i, j)$  belongs to trip  $k$  only if trip  $k$  was selected to be served by any garbage truck: constraint (9) assigns garbage truck  $w$  to arc  $(i, j)$ , constraint (10) ensures that arc  $(i, j)$  can be served by at most one garbage truck, and constraint (11) guarantees that the same garbage truck serves all arcs constituting trip  $k$ .

Constraint (12) ensures that capacity of garbage truck  $c$  cannot be exceeded. Garbage truck balance equations (13) and (14) determine arrival time at pickup node  $i$  of a garbage truck serving trip  $k$ . Constraints (15) and (16) ensure that time window defined for pickup node  $i$  and waste  $w$  is not exceeded, while constraints (17) and (18) guarantee that a garbage truck serving trip  $k$  leaves its home depot for collecting waste  $w$  and comes back during the time window which allows to pick up waste  $w$ . Constraint (19) ensures that a garbage truck serving trip  $k$  on which waste  $w$  is collected comes back to its home depot before the end of working hours.

Constraint (20) ensures that if a garbage truck serving trip  $k$  enters pickup node  $i$  then it must leave the node. Moreover, this constraint lets exactly one garbage truck collecting waste  $w$  visit pickup node  $i$ ; in other words, waste  $w$  is to be collected by at most one garbage truck.

Constraint (21) defines what type of solid waste is collected by a garbage truck serving trip  $k$ . Constraint (24) lets a garbage truck assigned to trip  $k$  serve arc  $(i, j)$  only if the garbage truck is allowed to traverse that arc, while constraint (25) lets garbage truck serving trip  $k$  collect solid waste  $w$  only if the garbage truck can handle that type of solid waste.

## COMPUTATIONAL EXPERIMENT

Computational experiment for a solution were executed on a computer with the Intel Core i7 quad-core CPU, running at 2.5 GHz in a Win10 OS, with 16 GB of RAM. The MIP model has

been solved, using the general-purpose mixed-integer optimization solver Gurobi, version 7.0.2.

We considered a selective waste collection system where three types of municipal solid waste are picked-up separately (*Bio*, *Seg*, and *Sol*) with a heterogeneous fleet consisting of two types of garbage trucks (*S<sub>v</sub>* and *L<sub>v</sub>*). The maximum number of trips to be set as the solution is limited to four {*tr<sub>1</sub>*, *tr<sub>2</sub>*, *tr<sub>3</sub>* and *tr<sub>4</sub>*}.

The waste collection system serves a network consisting of five pickup nodes {*N<sub>1</sub>*, *N<sub>2</sub>*, *N<sub>3</sub>*, *N<sub>4</sub>*, *N<sub>5</sub>*}, a depot {*N<sub>0</sub>*} and two specialized segregation units {*L<sub>I</sub>*, *L<sub>II</sub>*} (see Figure 3). From pickup nodes *N<sub>1</sub>*, ..., *N<sub>5</sub>* waste all three types is collected. Using the method presented in [Korcyl et al. 2016] we decompose node representing specialized segregation units *L<sub>I</sub>*, and *L<sub>II</sub>* into a set of apparent nodes (one for each trip *k*), so that the feasibility of the problem can be ensured. Therefore, in this instance specialized segregation units are substituted by sets of four nodes, respectively {*L<sub>0</sub>*, *L<sub>1</sub>*, *L<sub>2</sub>*, *L<sub>3</sub>*} and {*L<sub>4</sub>*, *L<sub>5</sub>*, *L<sub>6</sub>*, *L<sub>7</sub>*}; number four corresponds with the maximum number of trips to be set. If trip *k* is used in the system, one of nodes {*L<sub>0</sub>*, *L<sub>1</sub>*, *L<sub>2</sub>*, *L<sub>3</sub>*, *L<sub>4</sub>*, *L<sub>5</sub>*, *L<sub>6</sub>*, *L<sub>7</sub>*} must belong to it.

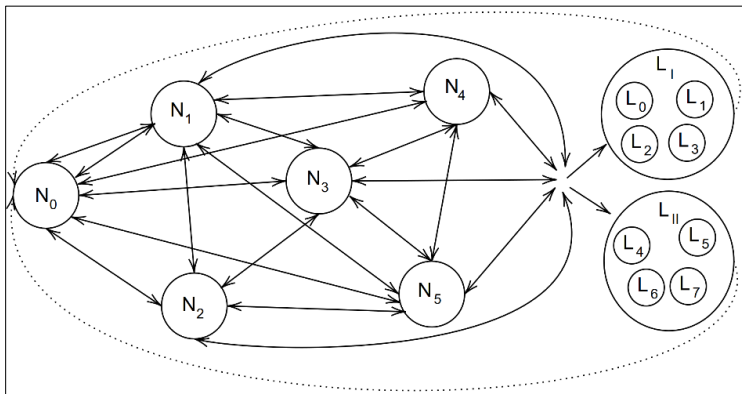


Fig. 3. Network served by the selective waste collection system.

Source: own work.

Demand  $d_{iw}$  for collection service of solid waste *w* in pickup node *i* is presented in Table 1. In Table 2 we present time windows ( $e_{iw}$ ,  $l_{iw}$ ) for picking up solid waste *w* from node *i*. Collection time of solid waste *w* from pickup node *i* ( $s_{iw}$ ) and the time window for collecting solid waste *w* ( $E_w$ ,  $L_w$ ) are presented in Table 3. We assumed that garbage truck *k* can traverse any arc and collect any type of waste. Times of traversing each arc are presented in Table 4. Travel times between any pickup node and any node representing the landfill are equal. Note that there is no arc between nodes *L<sub>0</sub>*, *L<sub>1</sub>*, *L<sub>2</sub>*, *L<sub>3</sub>*, *L<sub>4</sub>*, *L<sub>5</sub>*, *L<sub>6</sub>*, and *L<sub>7</sub>*.

In Table 5 we specified characteristic features of garbage truck of type *c*: the latest time of coming back to the home depot ( $p_c$ ), capacity ( $q_c$ ), and costs of using a garbage truck of type *c*: fixed cost ( $\alpha_c$ ), unit cost ( $\beta_c$ ) which depends on the length of the trip, and the cost of extra dwelling ( $\delta_c$ ). Value of coefficients  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  is 0.0001.

Table 1. Demand  $d_{iw}$  for waste collection service of waste *w* in pickup node *i*

	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$
<i>Bio</i>	6	10	3	2	3
<i>Seg</i>	4	8	8	1	1
<i>Sol</i>	6	9	4	5	6

Source: own work.

Table 2. Time windows for picking up selective solid waste *w* from node *i*

	The earliest time ( $e_{iw}$ )					The latest time ( $l_{iw}$ )				
	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$
<i>Bio</i>	13	19	15	20	12	92	63	76	100	84
<i>Seg</i>	28	13	1	5	4	51	79	73	87	90
<i>Sol</i>	23	35	18	32	10	82	79	57	69	54

Source: own work.

Table 3. Collection time of segregated solid waste *w* from pickup node *i* ( $s_{iw}$ ) and the time window for collecting solid waste *w* ( $E_w, L_w$ )

	Collection time ( $s_{iw}$ )					The earliest time	The latest time
	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$		
<i>Bio</i>	1	3	1	3	2	0	5
<i>Seg</i>	2	3	2	2	2	0	5
<i>Sol</i>	1	3	1	2	3	0	5

Source: own work.

Table 4. Time of traversing arc (*i, j*) ( $t_{ij}$ )

	$N_0$	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$	$\{L_0, L_1, L_2, L_3\}$	$\{L_4, L_5, L_6, L_7\}$
$N_0$	0	4	1	8	9	1	0	0
$N_1$	4	0	3	8	10	9	4	10
$N_2$	1	3	0	5	2	5	9	10
$N_3$	8	8	5	0	2	1	3	7
$N_4$	9	10	2	2	0	6	10	1
$N_5$	1	9	5	1	6	0	5	2
$\{L_0, L_1, L_2, L_3\}$	0	4	9	3	10	5	0	0
$\{L_4, L_5, L_6, L_7\}$	0	10	10	7	1	2	0	0

Source: own work.

Table 5. Specified characteristic features of waste collecting truck of type *c*

	$p_c$	$q_c$	$\alpha_c$	$\beta_c$	$\delta_c$
<i>Lv</i>	100	71	100	1	2
<i>Sv</i>	100	64	100	1	2

Source: own work.

## COMPUTATIONAL RESULTS

For the instance described in the previous section four trips were used for collecting waste of type *Bio*, *Seg* and *Sol* from all the pickup nodes (see Table 6). All the trips are served by garbage trucks of type *Lv*.

Table 6. Trips allocation for collecting different types of waste ( $y_{kw}$ )

	$tr_1$	$tr_2$	$tr_3$	$tr_4$
Bio	–	–	1	–
Seg	1	–	–	1
Sol	–	1	–	–

Source: own work.

In Table 7 and Table 8 arrival times and extra dwell time at pickup nodes is presented. In Figure 4 obtained schedule of selected solid waste collection is presented; it includes timelines for garbage trucks.

Table 7. Arrival time ( $a_{ik}$ ) and extra dwell time ( $u_{ik}$ ) on trip  $k$  in pickup node  $i$

	$N_0$	$N_1$	$N_2$	$N_3$	$N_4$	$N_5$
$tr_1$	0 (0)	–	–	8 (0)	–	11 (0)
$tr_2$	0 (5)	9 (22)	35 (0)	44 (0)	40 (0)	46 (0)
$tr_3$	0 (5)	9 (6)	19 (0)	29 (0)	24 (0)	31 (0)
$tr_4$	0 (5)	24 (4)	18 (0)	–	14 (0)	–

Source: own work.

Table 8. Arrival time ( $a_{ik}$ ) and extra dwell time ( $u_{ik}$ ) on trip  $k$  in Specialized sorting units

	$L_1$		$L_{II}$	
	$L_0$	$L_1, L_2, L_3$	$L_4, L_5$	$L_6, L_7$
$tr_1$	–	–	–	15 (0)
$tr_2$	–	–	–	51 (0)
$tr_3$	–	–	–	35 (0)
$tr_4$	34 (0)	–	–	–

Source: own work.

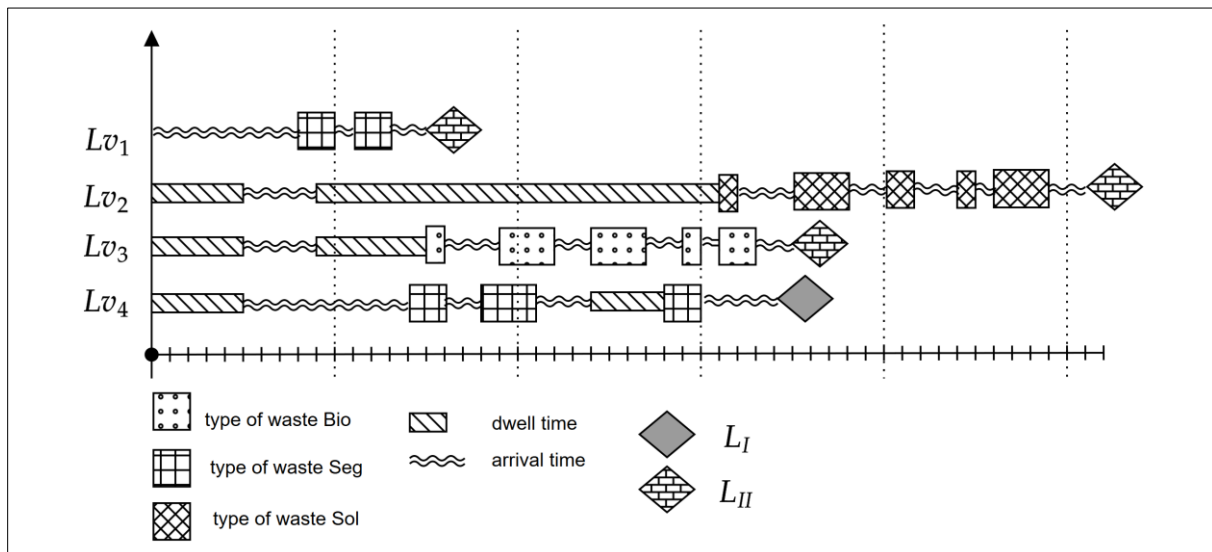


Fig. 4. Selected solid waste collection schedule.

Source: own work.

Computation time was limited to 30 sec. Feasible solution was found and objective function value was 30.43. Optimization GAP was 0.02% (optimization GAP is the difference between the value of the objective function computed for a given feasible solution and the best estimate of that value).

**CONCLUSIVE REMARKS**

The Fleet Optimization Problem for Selective Solid Waste Collection can be considered as a generic problem faced by SWM. Decisions of this kind belong to strategic planning and are made in SWM when solid waste collection schedule is being set and the fleet for solid waste collection service is being selected. Computation time is not a problem here and mathematical modeling is a proper method for solving these problems. Therefore, MIP model presented in this paper can be used as a decision-making support tool in SWM.

**ACKNOWLEDGEMENTS**

This work was supported by the AGH University of Science and Technology with the grant for the maintenance of research potential.

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